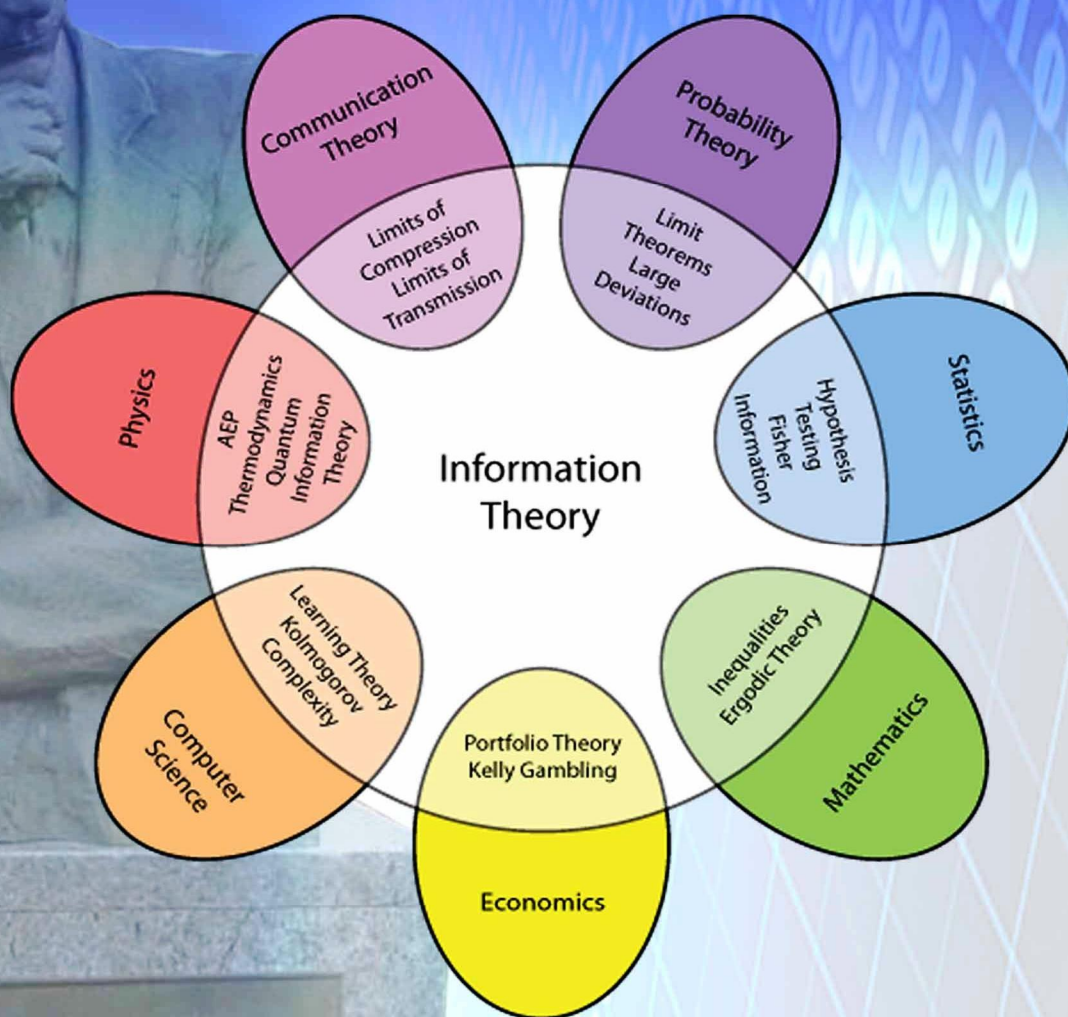
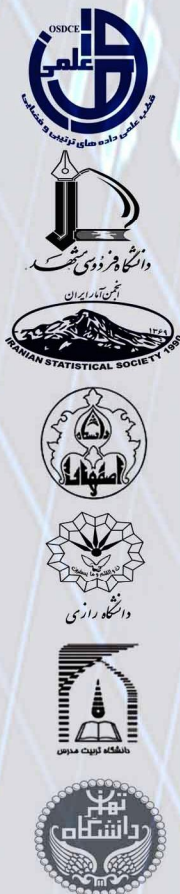


# The 2nd Workshop on Information Measures and Their Applications

Ordered and Spatial Data Center of Excellence  
Department of Statistics  
Ferdowsi University of Mashhad  
28-29, January, 2015



Claude Elwood Shannon  
Father of Information Theory



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In The Name of Allah



## Abstract of

# The 2nd Workshop on Information Measures and Their Applications

## Ordered and Spatial Data Center of Excellence

Department of Statistics  
Faculty of Mathematical Sciences  
Ferdowsi University of Mashhad

Mashhad, Iran

28-29 January, 2015

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# Preface

On behalf of the organizing and scientific committees, we would like to extend a very warm welcome to all the participants of the 2nd workshop on Information Measures and Their Applications. We hope this workshop provides an environment of useful discussions and would also exchange scientific ideas through opinions.

We wish to express our gratitude to the numerous individuals and organizations that have contributed to the success of this workshop, in which more than 60 colleagues, researchers, and postgraduate students have participated.

Finally, we would like to extend our sincere gratitude to the students of the Department of Statistics at Ferdowsi University of Mashhad for their kind cooperation. We wish them all the best.

Ordered and Spatial Data

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## Entropy of some integer valued stochastic processes

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### Abstract

In this paper, we consider the entropy and entropy rate of some integer valued stochastic processes. We illustrate and compare the entropy (rate) of thinned values of Poisson and geometric processes. Also, we study the entropy (rate) of integer valued AR(1) process with Poisson innovations.

**Keywords:** Entropy, Integer valued stochastic processes, Markov chain.



## Topics in maximum entropy modeling in continuous and discrete setting

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### Abstract

In this talk we address some new maximum entropy (ME) methods for modeling the distribution of time to an event in both continuous and discrete setting. In continuous case, after reviewing some of the existing results in the literature, we talk a new method which provides characterizations of change point models such as the piecewise exponential distribution. In discrete case it is proved by authors that within the class ultra log-concave distributions (of order  $n$ ) the Poisson (binomial) distribution is ME. We define a new class of discrete distributions called the weak ultra log-concave (WULC) distributions which includes the negative binomial distribution as boundary. We rise the conjecture that the negative binomial distribution is ME in the class of WULC distributions.

**Keywords:** Maximum entropy, Log-concave, WULC distributions.



## On robustness of entropy-based goodness of fit tests

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### Abstract

We study the robustness of a few entropy and classic goodness of fit tests. We investigate robustness of normality and exponentially tests with respect to heavy tailness and dependence. Non-Gaussian stable distributions are considered for the alternative hypotheses and exchangeability assumption is assumed under the null hypothesis. Our simulation results show that classical tests are more sensitive with respect to a few entropy tests for heavy tail data. However, the entropy and classical tests are robust with respect to the exchangeability assumption.

**Keywords:** Goodness-of-fit, Entropy, Power of test, Stable distribution.





## Monte Carlo comparison of entropy tests with energy for stable distributions

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### Abstract

In this paper, we introduce a statistic by the name of energy statistic. It is used to construct a goodness of fit tests. Also, we present an application of the energy statistic as a test for stable distribution. In certain cases, normal and Cauchy, we have been compared power of them with the entropy and classical tests. Our simulation results show that a test based on energy is better than a few entropy tests in some cases.

**Keywords:** Goodness-of-fit, Energy statistic, Entropy, Power of test.



## On entropy order for order statistics and their concomitants

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### Abstract

Let  $(X, Y)$  and  $(S, T)$  be two continuous random vectors. It is shown that if  $S$ ,  $[Y|X = x]$  and  $[T|S = x]$ , for all  $x$  are *DFR*,  $Y$  is stochastically increasing in  $X$  and  $(X, Y) \leq_{sst} (S, T)$ , then  $H(X, Y) \leq H(S, T)$ , where  $H(Z)$  is Shannon entropy of a random variable  $Z$ . Let  $(X_i, Y_i)$ ,  $i = 1, \dots, \max\{m, n\}$  be a set of independent copies of  $(X, Y)$ . It is also shown that if  $X$  and  $[Y|X = x]$ , for all  $x$  have *DFR* distributions and  $Y$  is stochastically increasing in  $X$ , then for  $i \leq j$  and  $n - i \geq m - j$ ,  $H(X_{i:n}, Y_{[i:n]}) \leq H(X_{j:m}, Y_{[j:m]})$ . Let  $(S_i, T_i)$ ,  $i = 1, \dots, \max\{m, n\}$  be a set of independent copies of  $(S, T)$ . It is observed that under certain set of mild conditions on  $F_{X,Y}$  and  $F_{S,T}$ , for  $i \leq j$  and  $n - i \geq m - j$ ,  $H(X_{i:n}, Y_{[i:n]}) \leq H(S_{j:m}, T_{[j:m]})$ . Finally, we discuss some conjectures about entropy properties of vector of order statistics corresponding to a random sample of size  $n$  from a symmetric distribution which admits density.

**Keywords:** Dispersive order, Decreasing failure rate, Strong stochastic order, Symmetric distribution.



## The appropriate model selection

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### Abstract

Statistical modeling is a crucial issue in scientific data analysis. Models are used to represent stochastic structures, predict future behaviour, and extract useful information from data. This paper examines how the model has been selected by model selection criteria. How the model is discussed by AIC and AICc, and compare these two criteria to criterion BIC is investigated. The information criteria AIC and AICc are studied in detail.

**Keywords:** AIC, Information Criterion, Model Selection.



## Entropy of distribution and relative entropy on Polish spaces

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### Abstract

In this paper, we assume  $\mu$  is a probability measure and the conditions for maximal entropy are provided. The properties of relative entropy for probability measure on Polish spaces are also discussed.

**Keywords:** Entropy, Uniform distribution, Geometric distribution, Product distribution.



## Quantile-based dynamic cumulative entropy

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### Abstract

Recently, it has been shown by many authors that quantile functions are efficient and equivalent alternatives to distribution functions in modelling and analysis of statistical data (Nair et al. 2013). In this talk, the quantile function is recalled and some reliability measures are rewritten in terms of quantile function. A quantile-based Shannon entropy function introduced by Sunoj and Sankaran (2012). Here, we consider the cumulative entropy (Rao et al. 2004, Asadi and Zohrevand, 2007) and obtain the quantile-based dynamic cumulative entropy (QDCE). Some properties of QDCE are presented.

**Keywords:** Entropy, Quantile, Characterization, Exponential distribution.



## Entropy maximization based on generalized Gini index

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### Abstract

In economics and social sciences, inequality measures such as Gini index, Peitra index etc., are commonly used to measure the evenness of probability distributions. In this paper, we first review some entropy maximization studies under moment and inequality measures constraints. Next, we consider a generalization of Gini index and based on the principle of maximum-entropy. We find the probability distribution that maximizes the entropy among all probability distributions supported on non-negative real values with a given mean and a given generalized Gini index.

**Keywords:** Maximum-entropy, Inequality measures, Euler's equation, Generalized Gini index.



## On dynamic mutual informations and cumulative Kullback-Leibler discriminations

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### Abstract

We consider dynamic mutual information of lifetime distributions and study this measure for bivariate past and residual lifetimes; some bounds are obtained and examples are given. We focus also on dynamic mutual information for TTE and truncated TTE models. The mutual information between the minimum and the maximum of order statistics is considered and a copula-based approach for this measure is presented. In the second part some properties of a new measure of discrimination obtained from Kullback-Leibler discrimination measure are described. A dynamic version of this measure is also proposed, and it is applied to some concepts of relative aging. Finally, we provide an application to image analysis.

**Keywords:** Entropy, Mutual information, Bivariate lifetimes, Time-transform exponential model.



## Parameter estimation by Kullback-Leibler divergence of Survival functions with application to censored data

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### Abstract

In this paper, we study estimation of parameters based on survival functions. We consider equilibrium distributions in Kullback-Leibler divergence and find a new measure of divergence. Then we use this measure in parameter estimation. Some extensions in discrete, censor and real numbers support cases also investigated.

**Keywords:** Censored data, Entropy, Estimation, Equilibrium distributions, Information measures.





## Entropy and information (divergence) measures

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The extension of notion for the measure of information with application in communication theory back to the experiences of the C. E. Shannon during the second World War II. In 1948, he introduced that the entropy is a real number associated with a random variable which is equal to the expected value of the surprise that we would receive upon getting a realization of the random variable. Let,  $S_a(p) = -\log_a p$  (base-2 is often used) be the measure of surprise we feel when an event with the probability  $p$  is occurring actually occurs. Then, entropy for a random variable is calculated using the probability mass (or pdf) function of the random variable via  $H_a(X) = E[S_a(p(X))]$ . Some of the properties and characterizations of the Shannon entropy and its extension versions are mentioned here.

Also, finding expressions for the multivariate distributions (discrete or continuous) and information measure such as mutual information with some of their properties and discussing in view of copula are reviewed.

The principle of maximum entropy provides a method to select the unknown pdf (or pmf) compatible to entropy under a specified constraint. This idea was introduced by Jaynes 1957 and obtained via a theorem by Kagan et al. (1973). Applying to maximum Renyi or Tsallis entropy and also  $\phi$ -entropy, as a general format subsume many special cases. Similar arguments are applicable to a multivariate set-up.

In probability theory and information theory, the Kullback Leibler divergence (also information divergence, information gain, relative entropy) is a non-symmetric measure of the difference between two probability distributions. Specifically, the Kullback Leibler divergence is typically represents the "true" distribution of data and a theoretical model for approximation of the true distribution. Although it is often intuited as a metric or distance, the KL divergence is not a true metric. Various applications in statistics and properties of it is one of our aim in here. The link between maximum likelihood and maximum entropy and Kullback Leibler information is important for a discussion which is coming in this note. There are several types of information divergence measure that are studied in literature as extensions of the Shannon entropy and Kullback Leibler information. Some of them can be collected in Csiszar  $\phi$ -divergence as special cases. So, minimization of them is important and finding these optimal measures is the other direction that is discussed in this paper with the related special states such as Kullback Leibler information,  $\chi^2$ -divergence, total variation, squared perimeter distance, Renyi divergence, Hellinger distance, directed divergence and so on.

**Keywords:** Entropy, Maximum entropy, Kullback Leibler information, Information measures, Minimization of Kullback Leibler information.



## A measure of relative entropy rate between two stochastic processes with an application in speech recognition

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### Abstract

In this paper, we study the relative entropy rate between a homogeneous Markov chain and a hidden Markov chain defined by observing the output of a discrete stochastic channel whose input is the finite state space homogeneous stationary Markov chain. For this purpose, we obtain the relative entropy between two finite subsequences of above mentioned chains and define the relative entropy rate between these stochastic processes, then calculate the maximum of the relative entropy rate by the concept of convexity and study the convergence of it.

**Keywords:** Relative entropy rate, Mutual information, Stochastic channel, Markov chain, Hidden Markov chain.



## Expression and bounds for the entropy of coherent system

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### Abstract

Entropy of engineering systems, as a measure of uncertainty, has been studied in statistical and reliability literature. In this paper, we provide an expression for the entropy of a coherent system lifetime by using the concept of minimal signature when lifetimes of components are independent and identically distributed. We also obtain bounds for the entropy of system lifetime in terms of the entropy of component lifetimes. It is shown that bounds are very useful when the system has a large number of components or the configuration of the system is complicated. Some examples are also given.

**Keywords:** Bridge system, Coherent system, KL information,  $k$ -out-of- $n$  system, Minimal signature.



## A short note on the cumulative residual entropy

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### Abstract

In this paper, we study maximum entropy approach in terms of Shannon entropy and entropy of equilibrium distribution, known as cumulative residual entropy (CRE). In classical maximum entropy approach the model whose uncertainty is maximum in a set of distributions, under some constraints (usually moments constraints), selected as the maximum entropy model. We present maximum entropy of equilibrium distribution model, under some partial ordering in well known family of lifetime distributions

**Keywords:** Uncertainty, Cumulative Residual Entropy, Equilibrium Distribution, Partial Ordering, Maximum Entropy.